**PART 2**: **EXPLAINING** **THE BEHAVIOUR OF FINANCIAL ASSET PRICES: INTEREST RATES AND EXCHANGE RATES**

CHAPTER 5:

Understanding Interest Rates:

Definitions and Concepts

**FOCUS OF THE CHAPTER**

This chapter provides a discussion of the definitions and concepts of various **rates of interest**. The chapter also discusses how to calculate the **present value** of various types of financial assets, and the rate of return (yield or interest rate) on these assets, using thepresent value concept. A discussion of **real and nominal interest rates** and their relationship to **inflation rates** and **tax** **rates** is also presented with the help of the **Fisher equation**.

**Learning Objectives:**

1. Define the interest rate
2. Define the present value concept
3. Describe the yield to maturity as an interest rate measure and explain why it is important
4. List the interest rate calculations associated with different payment schemes
5. Explain how the yield on simple loans, coupon bonds, zero coupon bonds, STRIPS, and other instruments is evaluated
6. Explain the real interest rate and why it is an important concept
7. Identify how strong the connection is between inflation and the nominal interest rate

**SECTION SUMMARIES**

**The Present Value Concept**

In this chapter we accept without question that financial markets exist and people have the option of buying financial instruments of various kinds and earning interest. That simple fact is responsible for giving money its “time value”. A dollar today is not worth the same as a dollar in the future simply because a dollar today can be invested or deposited in an interest paying account and will therefore grow to a larger amount in the future. We should say at the outset that in this chapter we completely ignore inflation and taxes and deal only with what we call nominal interest.

Suppose, for illustration purposes, a baby is born and each of its grandmothers decides to give the baby a cash gift. One grandmother gives the baby $50,000 and the day it is born and the other grandmother declares that she will give the child $100,000 on the day the child turns 18 years of age. How do we compare (even though it might be rude to do so) the generosity of the two gifts? We could compute the amount that the $50,000 gift would grow to if left on deposit earning interest for 18 years or we could compute how much has to be deposited today to grow to $100,000 in 18 years.

For discussion purposes, let us refer to the amount available today (i.e., $50,000) as a present value or PV for short and the future amount (i.e., $100,000 in 18 years as the future value or FVn. The n tells us that we are referring to the future value in n years. To determine the amount that $50,000 will grow to in 18 years we use the formula

FVn = PV ( 1+ r ) n 5.1

This equation assumes that compounding occurs once per period, once per year. If interest is compounded twice per year the formula is

FVn = PV ( 1+ r / 2 ) 2n 5.2

Or, in general for k times per year,

FVn = PV ( 1+ r / k ) kn 5.3

Using an interest rate of 6% compounded quarterly, $50,000 would grow to $146,057.90 in 18 years.

We could have asked a different question, namely, how much would we need to deposit in an interest bearing account in order to have $100,000 eighteen years from now? To find this, we simply need to solve equation 5.1 for the present value because here we presume that we know the future value, the interest rate and the number of periods. That would give us

PV = FVn / ( 1+ r ) n 5.4

Using an interest rate of 6% compounded quarterly, we would require a deposit of $34,233.00 in order to have $100,000.00 eighteen years from now. Clearly, $50,000 today is worth more than $100,000 18 years from now. When we are calculating present values we often refer to this as discounting, i.e., determining the discounted present value of a future sum.

Instead of compounding or discounting we could have a different question. We might want to know at what interest rate we would be indifferent between $50,000 today and $100,000 18 years from now. To solve this problem we simply solve equation 5.1 for r.

That would give us

r = ( FVn / PV ) 1/n – 1 5.7

Using n = 18 we find that the interest rate is 3.93%. We can use this formula anytime that we want to know what an investment is yielding.

Again, equation 5.7 assumes that compounding occurs once per period, once per year. If interest is compounded twice per year the formula is

r = 2 (( FVn / PV ) 1/2n – 1) 5.8

Or, in general for k times per year,

r = k (( FVn / PV ) 1/kn – 1) 5.9

In effect, we are asking what interest rate we would have to earn in order that a $50,000 would yield a payback of $100,000 in 18 years. We can use these formulas to calculate the yield on any type of investment for which there is a single future payback.

*Multiple payments* All possible applications of the previous formulas involve a single payment made and a single payment received. Many more real world problems involve a series of payments. People periodically deposit sums of money over several periods in order to have a large sum for a particular purpose such as retirement or to buy a large valued item. In other situations investments are made and several payments are received over a number of periods. Some of these situations will involve payments of irregular amounts perhaps sporadically made or received. These we cannot do much to simplify. We simply must apply on or more of the above formulas to suit the purpose. If, for example, payments were being made of irregular amounts we could determine the future value at some time in the future n years from now using the formula

FVn = C1 ( 1 + r ) (n-1) + C2 ( 1 + r ) (n-2) + C3 ( 1 + r ) (n-3) +. . . .+ Cn 5.10

Where Ci is the payment made at the end of each year i for n years and r is the interest rate over that period.

Alternatively, we might want to know the present value of a series of random payments and we could use a formula such as

PV = C1 / ( 1 + r ) + C2 /( 1 + r ) 2 + C3 / ( 1 + r ) 3 + . .+ Cn / ( 1 + r ) n 5.11

There are many other circumstances in which the amounts of the payments are equal and are received at regular intervals. Loan and mortgage payments, the coupon portion of bond payments and annuities are good examples. We could simply use equations 5.10 and 5.11 but that would involve many calculations. Fortunately, equation 5.10 can be simplified to

FVn = C [ ( 1 + r ) n - 1 ] / r where C is the regular payment. 5.12

Equation 5.11 can be simplified to

PV = C [ 1 - 1 / ( 1 + r ) n] / r where C is the regular payment. 5.13

In the event that payments are made or received and the beginning rather than the end of the period, the above equations should be modified as follows:

FVn = C ( 1 + r )[ ( 1 + r ) n - 1 ] / r where C is the regular payment. 5.14

PV = C1 ( 1 + r )[ 1 - 1 / ( 1 + r ) n] / r where C is the regular payment. 5.15

To calculate the present value of a bond, say a $10,000 10 year bond with coupon rate of 4% when the market rate is 5%, we use equation 10.1 to determine the present value of the final principle payment, i.e.,

PV = FVn / ( 1+ r ) n = $10,000 / ( 1 + .05 ) 10 = $6139.13 5.16

and we use equation 5.13 to determine the present value of the coupon payments, i.e.,

PV = C [ 1 - 1 / ( 1 + r ) n] / r = $ 400 [ 1 - 1 / ( 1 + .05 ) 10 ] / 0.05 = $ 3088.69

The total present value of the bond is the sum of these two amounts or $9,227.82.

*Market rates of interest and asset prices:* In the previous example we had two interest rates: a contracted rate which we called the coupon rate which we use to determine the periodic payment and a market rate which we used as r in the present value formula. For the moment we are assuming that we will be able to find out the interest rate or effectively the yield on comparable securities. To be comparable securities should have the same approximate risk, maturity and tax treatment. We could discover this rate by examining the daily financial press. When we use the market rate of interest to calculate present value we determine the market value of the security. If we were selling that security, other people would be willing to pay, at most, that price because in doing so they will in fact earn the market rate of interest.

# **Perpetuities**L There are many circumstances in which assets generate a regular cash flow for an indefinitely long period of time. In fact, at normal interest rates the discounted present value of funds that are to be paid or received more than thirty or forty years into the future are relatively insignificant. It is, therefore useful to have a formula for an infinite stream of earnings of a regular amount. If we observe that if n goes to infinity in the equation PV = C [ 1 - 1 / ( 1 + r ) n] / r this equation will simplify to

PV = C / r 5.17

## *Yields and the internal rate of return*: When we developed equation 5.7 above we simply solved equation 5.1 for the interest rate r. We could not easily do that for equations 5.11 or 5.13. In order to find that particular interest rate, or the yield, we have to find that rate at which the discounted present value of the future cash flows is approximately equal to the present or market value. Computers and financial calculators will do this for us but if we have to do it ourselves we use a trial and error or iterative method. We pick an interest rate and calculate the present value. If the answer we get is larger than the market value we recalculate present value with a higher interest rate. We continue to adjust the interest rate until the discounted present value and the market value are approximately equal. The interest rate at which this equality holds is the yield on the security. Yield and internal rate of return are essentially equivalent.

***Debt contracts:*** Many debt contracts require equal monthly or annual payments of blended principle and interest. To determine the amount of the annual payment we simply solve equation 5.13 for the C value using PV as the amount of the initial loan. C will be the amount of the annual payment.

C = PV / {[ 1 - 1 / ( 1 + r ) n] / r} 5.18

There are many more formulas that we could develop to fit various circumstances but students are advised to develop their own equations or financial spreadsheets to solve the problems at hand.

**Holding Period Yield;** Financial instruments are not always held until maturity. They may be sold before the date of maturity. The holding period yield is the rate of return for the period which the instrument is held. The financial instrument may have been purchased at a discount or at a premium or may be sold at a price different from the purchase price. The difference between the selling price and the purchase price accounts for the capital gain or loss from holding the asset. In calculating the holding period yield both the current yield and the capital gain or loss return rate are taken into account.

Holding Period Yield = Current Yield + Capital Gain or Loss Return Rate

Holding Period Yield(i) = (C/PV) + ( ΔPV/PV)

where C is coupon payment, PV is present value, Δ PV is capital gain or loss.

Bond prices are more volatile (i.e., the magnitude of the change in bond prices in response to a change in the interest rate is larger) at longer terms to maturity than at shorter terms to maturity.

**The Distinction between Nominal and Real Interest Rates**

Inflation, taxes, and expectations about them have a considerable effect on interest rates.

***The Role of Inflation Expectations:*** Inflation reduces the purchasing power of money. Lenders (buyers of financial assets), in an attempt to protect themselves from a loss of purchasing power, form expectations about future inflation and incorporate these expectations into the interest rate (the price of debt). On the one hand, the nominal interest rate (the rate of interest in current dollar terms) includes the compensation for postponed current consumption of the lender (called the real interest rate). On the other hand, it includes compensation for the expected loss of purchasing power due to inflation.

*The Fisher Equation:* The Fisher equation (named after Irving Fisher) states that the nominal interest rate (R) is the sum of the real interest rate (ρ) and the expected rate of inflation (πe) (i.e., R = ρ + πe, or ρ = R - πe). Using actual inflation (π) in place of the expected rate of inflation, one can calculate the *ex-post* real interest rate. Using a measure of the expected rate of inflation, the *ex-ante* real interest rate can be calculated.

*Some Historical Data:* During the last 40 years, the rate of inflation and the nominal interest rate appear to have moved together in the same direction. Real interest rates have varied considerably over time. There have been three phases of real interest rate behaviour. Real interest rates were stable in the 1960s, volatile during the 1970s and 1980s, and stable again in the 1990s. Also, in the industrial world, the rate of inflation and nominal interest rates appear to have moved together in the same direction, during the period from 1980 to 1999.

*Empirical Evidence:* In the long run, factors other than inflation, such as the overall productivity of the economy, can influence the real interest rate. Nevertheless, many empirical studies have found a negative relationship between inflation and *ex-post* real interest rates, although the exact cause of the relationship is unclear.

***Taxes:*** Real interest income is subject to income tax in Canada. Therefore, the nominal interest after taxes (R\*) is given by: R\* = R(1-τ), where τ is the income tax rate. The *ex-ante* real interest after tax can be obtained as:

*Ex-ante* real interest rate after tax = R\* - πe , or

*Ex-ante* real interest rate after tax = R(1-τ) - πe

**MULTIPLE-CHOICE QUESTIONS**

1. The price of renting money is

a) the purchasing power of money.

b) the purchase price of a bond.

c) the rate of interest.

d) the rate of inflation.

2. The discounted value of a stream of future payments is called

a) the present value.

b) the future value.

c) the time value of money.

d) the yield to maturity.

3. The rate of interest is 5%. The present value of $2100 to be received a year from today is

a) $5025.

b) $1000.

c) $2000.

d) $2500.

4. The present value of a given stream of payments increases as

a) the rate of interest increases.

b) the rate of inflation increases.

c) the tax rate decreases.

d) the rate of interest decreases.

5. For a simple one-year loan, the yield to maturity

a) cannot be calculated.

b) and the simple rate of interest are the same.

c) is greater than the current yield.

d) is less than the simple interest rate.

6. Compounding refers to

a) the process of adding interest payments on the interest payments already earned.

b) the process of calculating the present value of a financial asset.

c) the process of calculating the yield to maturity on a bond.

d) the process of amortization.

7. Increases in the market value of an asset held are called

a) capital gains.

b) leverage.

c) equity.

d) net worth.

8. According to the *ex-ante* version of the Fisher equation, the nominal interest rate is

a) the difference between the expected rate of inflation and the real interest rate.

b) the sum of the expected rate of inflation and the real interest rate.

c) the difference between the actual rate of inflation and the real interest rate.

d) the sum of the actual rate of inflation and the real interest rate.

9. If the income tax rate is 20%, the nominal rate of interest is 10%, and the expected rate of inflation is 3%, the *ex-ante* real interest rate after tax

a) cannot be calculated from the data given.

b) is 7%.

c) is 17%.

d) is 5%.

10. During the last four decades, the rate of inflation and the nominal interest rate in Canada

a) have moved in opposite directions.

b) have steadily increased over time.

c) have steadily decreased over time.

d) have moved together in the same direction.

## PROBLEMS

1. Assuming an interest rate of 5%, calculate for the following assets

a) the present value of a bond paying $100 in interest per year for two years, to be redeemed for $2000 at the end of two years.

b) the future value, at the end of three years, of a $1000 loan issued today.

c) the present value of a bond which pays an annual coupon of $250 forever.

2. Suppose you borrowed C$10,000 today at a 10% annual rate of interest for three years. How much is the annual payment you must make if you want to pay off the debt in three equal annual payments?

3. a) If the interest rate is 8% per annum, find the present value of a security that has a payment of $1100 in one year, $1200 in two years, and $1400 in three years.

b) If the asset mentioned above is sold for $3000, is the yield to maturity more or less than 8%? Why?

4. The consumer price index (CPI) increased from 200 to 210 between 2000 and 2001. People expect the price index to decrease by 4% in 2002. The tax rate is expected to remain at 20%. The nominal rate of interest in 2001 was 15 percent and the after-tax real rate of interest in 2002 is expected to remain the same as in 2001.

a) Calculate the after-tax real interest rate for 2001.

b) What is the *ex-ante* after tax nominal interest rate in 2002?

5. Determine the market value of a 10 year, $10,000 bond with an annual coupon payment of 5%

a) if the market rate of interest is 4%.

b) if the market rate of interest is 6%.

6. Suppose you bought the bond described in question 5 when the interest rate was 4% and the following day interest rates rose to 6%. Should you sell the bond and reinvest in a bond yielding 6%? Explain with reference to the values calculated in question 5.

7. Suppose that once you have become rich and famous you decide to endow a scholarship in your name that will pay $1,000 to a student every year for an indefinitely long period of time. What amount would you have to deposit to fulfil this obligation if the interest rate is 4%?

**ANSWER SECTION**

**Answers to multiple-choice questions:**

1. c (see page 72)
2. a (see page 72)
3. c (see pages 72-73)
4. d (see page 73)
5. b (see page 73)
6. a (see page 75)
7. a (see page 79-80)
8. b (see page 82-83)
9. d (see page 85-86)
10. d (see pages 84-85)

**Answers to problems:**

1.a) The present value (PV) of the bond can be calculated as follows:

PV = 100/(1+0.05) + 100/(1+0.05)2 + 2000/(1+0.05)2

= 100/(1.05) + 100/(1.1025) + 2000/(1.1025)

= 100(0.952381) + 100(0.9070) + 2000(0.9070)

= 95.2381 + 90.7000 + 1814.0000

= 1999.9381

b) The future value (FV) of the loan can be calculated as follows:

FV = 1000(1+0.05)3

= 1000(1.157625)

= 1157.625

c) The present value (PV) of this bond (perpetuity) can be calculated as follows:

PV = 250/0.05 = 5000

2. The fixed annual payment (FP) can be found by solving for FP in the following equation:

10,000 = FP/(1+0.10) + FP/(1+0.10)2 + FP/(1+0.10)3

10,000 = FP[1/(1+0.10) + 1/(1+0.10)2 + 1/(1+0.10)3]

10,000 = FP[0.909+0.826+0.751]

10,000 = FP[2.486]

10,000/[2.486] = FP

4022.5261 = FP

3 .a) PV = 1100/(1+0.08) + 1200/(1+0.08)2 + 1400/(1+0.08)3

= 1100/(1.08) + 1200/(1.1664) + 1400/(1.2597)

= 1018.5185 + 1028.8066 + 1111.3757

= 3158.7008

b) The yield to maturity on this bond is greater than 8%, because the purchase price of the bond ($3000) is lower than its present value as calculated above (i.e., $3158.7008).

4. a) The actual inflation rate (π) in 2001 can be calculated using the values of the consumer price index as follows:

π = (CPI2001 -CPI 2000 )/CPI1999 = (210-200)/200 = 10/200 = 0.05 or 5%

Now, the *ex-post* after-tax real interest rate (ρ\*) for 2001 can be calculated as follows:

ρ\* = R(1-τ) - π = 15(1-0.2) - 5 = 15(.8) - 5 = 12.0 - 5 = 7 (i.e., ρ\* = 7%)

b) The *ex-ante* before-tax nominal rate of interest (R) can be found by solving the following equation for R, given that ρ\* is expected to remain at 7%:

ρ\* = R(1-τ) - πe = 7

R(1-0.2) - (-4 ) = 7

0.8R + 4 = 7

0.8R = 7 - 4

R = 3/0.8 = 3.7 (i.e., 3.75%)

5. a) PV of face amount = 10,000/(1+.04)10 = 6,755.64

PV of coupons = 500[1- 1/(1+.04)10 ](1/.04) = 4,055.45

Total market value = 10,811.09

b) PV of face amount = 10,000/(1+.06)10 = 5,583.95

PV of coupons = 500[1- 1/(1+.06)10 ](1/.06) = 3,680.04

Total market value = 9,263.99

6. If you bought the bond when interest rates were 4% you would have paid $10,811.09. A day later, when interest rates are 6%, the market value of the bond is only $9,263.99. Since this amount reflects a yield to maturity of 6%, you gain nothing by selling the bond and repurchasing another one.

7. This is a perpetuity. You will need to deposit $1,000 / 0.04 or $25,000.