# ***Engineering Economics – Chapter 2***

# Alternate method when g > i

If g > *i*, then g1 will be negative. The factor could still be calculated with the formula but it cannot be obtained from the table. A different form can be used in this case.



Define a fictitious growth rate:

 (11)

Note: Although the factor is F/A, here it is used to calculate a present value.

Example: A series of 10 transactions grows by 9.2% each time, starting at $100. If *i* = 5% per period, find PV (Present Value).

(a) Method 1 - **eqn. (10):** 



(b) Method 2 **eqn. (11):** 



Note: One method has (1 + g) in the denominator, whereas the other has (1 + *i*) in the denominator.

Example: A person's salary is increasing at 5%/year and the interest rate is 5%/year. With an initial after-tax salary of $20,000 received in one year, what is the present value of the person's earnings if he/she works for 25 years.



= $20,000, *i* = 0.05, g = 0.05, n = 25

case: *i* = g → P = 20,000(25)/1.05 = $476,190

Examples of equivalence calculations

### F

### P

### A

### G

### g

Composite Cash flows:

- can apply factors to subsets of transactions

1. Negative Linear gradient

An investor can make 3 annual end-of-year payments, starting with $15,000 but reduced by $1,000 each year thereafter. The project will generate receipts of $10,000 at the end of year 4 which will increase annually by $2,500 for the following 4 years. If the investor can earn a rate of return of 10 percent annually, is this alternative attractive?

PW = [-15,000 – (-1000)(A/G, 10, 3)](P/A, 10, 3) + [10,000 + 2500(A/G, 10, 5)](P/A, 10, 5)(P/F, 10, 3)

$10,000

$12,500

$15,000

$20,000

$17,500

$15,000

$14,000

$13,000

8

7

6

5

4

3

2

1

Example: An ambitious saver plans to deposit $2,000 in a money market account starting 1 year from now and wants to increase annual deposit by $1,000 each year for the following 6 years. Assuming that deposits earn 9 percent annually, determine what equal-payment annuity would accumulate the same amount over the 7-year period.



0

3

2

4

5

6

1

3G

2G

G

5G

4G

6G

A = + G (A/G, *i*, N)



= $2,000 + $1,000 (A/G, 9, 7) = $2,000 + $1,000(2.6574) = $2,000 + $2,657 = $4,654

=> seven equal payments of $4657 are equivalent to seven payments increasing by $1,000 from $2,000 for the first one to $8,000 for the last one.

PW = [-15,000 + 1,000(0.9366)] x 2.4896 + [10,000 +2,500(1.8101)] x 3.7908 x 0.7513 = 6,356.08

Conclusion: positive PW, therefore invest.

2. Negative geometric gradient:

You win a lottery which pays $20,000 and 20 end-of-year payments for an amount equal to 5% LESS than the previous year. If you save all of the money, and the interest rate is 7% how much will you have at the end of 20 years.

Let 

F = [20,000 +A/(1+g)\*(P/A, 12.63, 20)] x (F/P, 7, 20) = [20,000 + 19,000/(0.95) x (7.184)] x (3.8697)

F = $633,392

Would you rather have $150,000 now?

Can this problem be done without separating the initial receipt of $20,000 from the rest of the cash flow?



3. Composite cash flow

An 18-year old student will need to withdraw $5,000 per year for 6 years for her education starting in 1 year (Master's degree included). Her rich uncle will contribute $3,000 into the account now, increasing by a fixed amount each year until the end of the 5th year. What is the required annual gradient to exactly satisfy the student's requirements? *i* = 12%/year.

Set the PW equal

5000(P/A, 12, 6) = [3000(P/A, 12, 6) + G(P/G, 12, 6)](F/P, 12, 6)

5000(4.1114) = [3000(4.1114) + G(8.9302)](1.12)

G = $674.15/year

Categories of unknowns

For most calculations involving factors, there are 4 variables of which 3 are known and the fourth one can be solved for. For example, given: P, *i*, N solve F. Sometimes *i* or N is the unknown.

Example: Invest $1,000 for 6 years to attain $1,600. What rate of return *i* is required?

F = P(F/P, *i*, N) → (F/P, *i*, 6) = F/P = 1.6 = (1 + *i*)6 → 1 + *i* = (1.6)1/6 = 1.08148 → *i* = 0.08148 = 8.15%/yr

or by interpolation in the interest tables.

(F/P, 8, 6) = 1.5868 and (F/P, 9, 6) = 1.6770 → *i* = 0.08 + 0.01  = 0.08146

IF THE EXPRESSION IS TOO COMPLEX TO ISOLATE *i*, THEN TRIAL & ERROR AND INTERPLATION MUST BE USED (OR A SUITABLE COMPUTER PROGRAM).

**EQUIVALENCE**

**Each Payment = $317.70**

$1,791

$1,000

**Each Payment = $237.40**

***i* = 6%**

0

1

2

3

4

5

6

7

8

9

10

#### Time in Years

• $1,000 today is equivalent to $1,791 received 10 years from now.

• $1,000 today is equivalent to $237.40 received at the end of each year for the next five years.

• $1,000 today is equivalent to $317.70 received at the end of years 6, 7, 8, 9 and 10.



4.2124 0.7473

• $237.40 received at the end of each year for the next 5 years is equivalent to a lump sum of $1,791 received 10 years from now.

• $317.70 received at the end of years 6, 7, 8, 9 and 10 is equivalent to $1,791 in 10 years form now.

• $237.40 received at the end of each year for the next 5 years is equivalent to $317.70 at the end of years 6, 7, 8, 9 and 10.

Example: Given the values of F and PA below, if *i* = 10%, find N?

P = 1000 F = 1600 *i* = 10% N = ?

(F/P, 10, N) = F/P = 1.6 = (1 + i)N = 1.1N → ln(1.6) = Nln(1.1) →  years.

or by interpolation in the interest tables.

(F/P, 10, 4) = 1.4641 and (F/P, 10, 5) = 1.6105 → N = 4 + 1  years.