#### *Engineering Economics – Chapter 4*

#### PROJECT BALANCE (Another view of the economic value of a project)

Concept: the profile of the outstanding balance for a project at every period.

Example: A $1,000 investment now generates $50 revenue after 1 year, and the revenue increases then by $500 per year for 4 years. The MARR is 10%.

+1972.95

The cash flow diagram is shown below.

500/year

+384.50

1,500

50

0

4

-605

3

2

-1000

-1000

1

-1050

The project balance is $1,000 at the start. At 10% interest, $1,100 is due at the end of year 1, but there is also a cash flow of $50 at that time. So the balance is -$1,050. Similarly, we find the balance at each period. Note that the balance at any point in time is the future value of all transactions to that point.

This evaluation criterion is very useful for several reasons:

(a) it shows the net future value for the project;

(b) the discounted payback period;

(c) the duration and magnitude of the risk of loss;

(d) eventual profit rate.

#### EXTRAPOLATIONS OF ANNUAL EQUIVALENT WORTH

A. Capitalized equivalent

The present value of a periodic series of cash flows that repeat indefinitely. This is applied particularly to very long term investments, or for endowment funds.



CE = A/i

Example: A new bridge will cost $10,000,000 with annual maintenance costs of $50,000 for the first 10 years, and maintenance costs of $150,000 per year after that. Find the capitalized equivalent (MARR = 8%)



CE = $11,204,000

So the actual cost of the bridge will be about 12% more including maintenance.

B. Capital Recovery Cost

A common decision in engineering economics is the purchase of capital equipment, possible value at the end of its service life. It is often useful to calculate the equivalent annual cost to be able to compare options.

Let: P be the initial cost

S the salvage value

CR = (P(A/P, i, n) – S(A/F, i, n) where costs are positive

But (A/F, i, n) = 

= -i + (A/P, i, n)

therefore: CR = (P-S)(A/P, i, n) + Si

The two formulae are equivalent, so we can use the one or the other. However, the second form is very common. As well, this form shows that the equivalent annual cost is due to the loss of value of the equipment over its lifetime (P-S), and the loss of interest Si each year on the salvage value which is not received until he end.

Example: We buy a machine for $10,000, which we expect to resell after 10 years for $1,000. What is the equivalent annual cost? (i = 12%).

CR = (10,000 – 1,000)(A/P, 12, 10) + 1,000(0.12)

CR = 9,000(0.1770) + 120 = $1,713

C. EAW (Equivalent Annual Worth) for Repeating Cycles

If a set of cash flows recurrs exactly after a fixed cycle, the AW for all of the cash flows is the same as the AW for cash flows within a single cycle.

D. EAW for Unit Pricing

Annual worth method is particularly useful if we must determine a cost or price per unit of usage. See examples in the textbook.

**OPTIONS WITH DIFFERENT LIFETIMES**

Lifetime of a capital good

- service life: how long is the equipment expected to physically operate?

- book life: based depreciation

- economic life: period that maximizes return

- perpetual life: infinite annuity, or very long life project

It is essential that options be compared over the same service life.

When the (lives) are different, we evaluate the options over some study period. The selection of a suitable study period can be based on:

(a) a standard study period set by the company;

(b) a period over which we can estimate the cash;

(c) a period which coincides with the lifetime of one of the options;

(d) any other reasonable period selected by the analyst.

Given the study period, we must adjust the calculations for the options whose lifetimes don't match. Let n be the length of the study period selected.

Example: A factory is reviewing its material handling system. They have three choices:

1. Do-nothing alternative (the alternative to a single alternative or project) Annual cost of manual handling: $9,200

Mutually exclusive alternatives: several alternative which fulfill the same need; selecting one excludes the other.

2. Construct a custom-made system, with a service life of 10 years

Initial cost: $15,000

Labour: $ 3,300/year

Hydro: $ 400/year

$6,400/year

Maintenance: $ 2,400/year

Taxes and Insurance $ 300/year

2. Buy standard automatic material handling, with a service life of 15 years

Initial cost: $25,000

Labour: $ 1,450/year

Hydro: $ 600/year

$5,625/year

Maintenance: $ 3,075/year

Taxes and Insurance $ 500/year

Which is the best option if MARR = 9%?

Case A – Study period of 10 years chosen

OPTION (1) P1 = 9,200(P/A, 9, 10) = 9200(6.4176) = $59,042

OPTION (2) P2 = 15,000 + (3300 + 400 + 2400 + 300) (P/A, 9, 10)

P2 = 15,000 + 6400(6.4176) = $56,073

OPTION (3) Its service life is LONGER than the study period: The main point here is that there is still value in the investment after the study period. If we can estimate a salvage value (i.e. resell price) for option 3 after 10 years we simply include it. (For example, if we assume that this system will be worth SV = $5,000 after 10 years, the present value is calculated as follows:

P3 = 25,000 + (14500 + 600 + 3076 + 500)(P/A, 9, 10) – 5,000(P/A, 9, 10)

P3 = 25,000 + (5625)(6.4176) – 5000(0.4224) = $58,987

N.B. the costs have been treated as positive, so we choose the smallest PV is OPTION (2).

We could have just as easily calculated the EAW (equivalent annual worth) for each option over 10 years:

EAW1 = P1 (A/P, 9, 10) = $59,042(0.1558) = $9,200/year

EAW2 = P2 (A/P, 9, 10) = $56,073(0.1558) = $8,736/year

= 15,000 (A/P, 9, 10) + 6400 = $8,737

EAW3 = P3 (A/P, 9, 10) = $58,987(0.1558) = $9,190/year

= 25,000 (A/P, 9, 10) + 5625 =

IMPLIED SALVAGE VALUE APPROACH: If we can't estimate a good S, we can instead calculate the capital recovery cost of the investment over its lifetime (i.e. C.R.), and simply ignore the portion which occurs after the study period, and also ignore the O & M which occur after the study period. CR = P(A/P, i, n) – S(A/F, i, n)

Example: for option 3, the CR for 15 years is given by:

CR3 = 25,000 (A/P, 9, 15) = $3,102,50/year

(0.1241)

Adding the annual O&M: $3102.50 + $5625 = $8727.50/year

According to this calculation, option (3) is marginally the preferred choice.

This method assumes that the value remaining in the equipment is represented by the annuities in years 11 to 15 (that is, the CR portion that we have ignored).

In effect, there is an implied salvage value at the end of the 10 years, which gives the same CR over the 10-year study period, as the equipment would have over its service life of 15 years.

So, we can calculate the implied SV which would give the same annual cost:

[CR = P(A/P, i, n) – S(A/F, i, n)]

$3102.50 = $25,000 (A/P, 9, 10) – Sn (A/F, 9, 10) = $12,044

(0.1558) (0.0658)

## Case B Study period of 15 years chosen

For option 2, we now have a lifetime shorter than the study period. We must make an assumption about how the need is satisfied for the cost of the study period once we are past the service life. We look at 2 methods of doing this.

Method I: Directly include the cost of replacing service once its useful life is passed.

Example: Suppose that the option (2), we revert to manual handling for the final five years.

EAW2 = P2(A/P, 9, 15) + 9200 (F/A, 9, 5)(A/P, 9, 15)

= 56,073(0.1241) + 9200 (5.985)(0.034) = $8,836

Conclusion: we choose option (3) where EAW3 = $8,727

Method II: The lowest common multiple.

We assume that each machine may be replaced by another identical one (with all of the same costs) at the end of its service life, and we select a study period equal to the lowest common multiple of the lifetimes of all options. For the example above, n = 30 years.

The calculations are simple, because EAW over one life cycle is the same as EAW over several repetitions. For Option (1) it is simply $9,200/year, for Option (2) it is the EAW over 10 years (which we have already calculated; and for Option (3), it is the EAW over its lifetime of 15 years (which we have also calculated already).

Conclusion: We are essentially indifferent between Options (2) and (3). In case, consider intangibles.

N.B. The calculations of EAW for the method of the least-common-multiple gives the same numerical answer as the implied salvage value approach (although the justifications are different). Therefore, this calculations is very common in practice.

##### Choosing Amongst Options

Example: Consider the following plans:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | End-of-year profit | | |
| Plan | Initial Cost | Year 1 | Year 2 | Year 3 |
| I | -1000 | 550 | 550 | 550 |
| II | -2000 | 875 | 875 | 875 |
| III | -3000 | 1400 | 1400 | 1400 |
| IV | -4000 | 1665 | 1665 | 1665 |

Also, there is a $5,000 budget which cannot be exceeded. Plans I and II are mutually exclusive, and Plan IV cannot be undertaken unless Plan I is selected. MARR = 15%

These constraints lead to the following mutually exclusive and collectively exhaustive feasible options:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Options | Plans | | | | | End-of-year Transactions | | | | PV @ 15% | IRR% |
| I | II | III | IV | **0** | | **1** | **2** | **3** |
| 1 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | MARR |
| 2 | 1 | 0 | 0 | 0 | -1000 | | 550 | 550 | 550 | 255.8 | 29.9 |
| 3 | 0 | 1 | 0 | 0 | -2000 | | 875 | 875 | 875 | -2.2 | 15.0 |
| 4 | 0 | 0 | 1 | 0 | -3000 | | 1400 | 1400 | 1400 | 196.5 | 18.8 |
| 5 | 1 | 0 | 1 | 0 | -4000 | | 1950 | 1950 | 1950 | 452.3 | 21.7 |
| 6 | 1 | 0 | 0 | 1 | -5000 | | 2215 | 2215 | 2215 | 257.3 | 15.7 |
| 7 | 0 | 1 | 1 | 0 | -5000 | | 2275 | 2275 | 2275 | 194.3 | 17.3 |

## DIFFERENCES BETWEEN OPTIONS

The fundamental rule for selecting the best option from alternatives A1 and A2, is that the evaluation criteria (PV, IRR, AE, or FV) is applied to the difference between the cash flow for the two options (i.e. A2-A1). This ensures that each additional investment amount earns interest at an acceptable rate (i.e., > MARR)

To apply the rule

(a) List the (mutually exclusive) options in increasing order of initial cost.

(b) The cheapest option becomes the defender.

(c) The next cheapest option becomes the challenger. Apply an appropriate criterion to the cash flows difference between the two options. If the incremental investment is desirable, the challenger becomes the defender. Otherwise, the challenger gets rejected.

(d) Continue this process until all the options have been evaluated. The last remaining defender is the best economic choice.

The method leads to the choice of the largest possible investment where every increment earns at least the MARR!!

Present Value based on incremental investment

A. defender 1 challenger 2 differences in transactions

-1000 550 550 550

(-1000-0) (550-0) (550-0) (550-0)

PV @ 15% = 255.80 > 0 -1000 + 550 (P/A, 15, 3) = 255.76

(2.2832)

Therefore option 2 becomes the defender

B. defender 2 challenger 3 differences in transactions

-1000 325 325 325

[-2000-(-1000)] [875-550]

PV @ 15% = -258 < 0 -1000 + 325 (P/A, 15, 3) = -257.96

(2.2832)

Therefore option 2 remains the defender

C. defender 2 challenger 4 differences in transactions

-2000 850 850 850

[-3000-(-1000)] [1400-550]

PV @ 15% = -59 < 0 Therefore option 2 becomes the defender (option 4 is rejected)

D. defender 2 challenger 5 differences in transactions

-3000 1400 1400 1400

[-4000-(-1000)] [1950-550]

PV @ 15% = 196.5 > 0 Therefore option 5 becomes the defender

E. defender 5 challenger 6 differences in transactions

-1000 265 265 265

[-5000-(-4000)] [2215-1950]

PV @ 15% = -365 < 0 Therefore option 5 remains the defender (option 6 is rejected)

F. defender 5 challenger 7 differences in transactions

-1000 325 325 325

[-5000-(-4000)] [2275-1950]

PV @ 15% = -258 < 0 Therefore option 5 remains the defender (option 7 is rejected)

Conclusion: the best choice is option 5, which means that we invest in Plans I and III.

**(Note** the PW method can be applied to each option individually, and the answer will be the same as that resulting from the incremental PW analysis**)**

Rate of Return based on incremental investment

A. defender 1 challenger 2 differences in transactions

-1000 550 550 550

IRR = 29.9% > MARR Therefore option 2 becomes the defender

0 = -1000 + 550 (P/A, i, 3) → (P/A, i, 3) =  = 1.8182 = 

Let x = (1 + i)

1.8182 = → 1.8182x4 – 1.7172x3 = x3 – 1 → 1.8182x4 – 2.8182x3 + 1 = 0

B. defender 2 challenger 3 differences in transactions

-1000 325 325 325

IRR = -1.3% < MARR Therefore option 2 remains the defender (option 3 is rejected)

0 = -1000 + 325 (P/A, i, 3) → (P/A, i, 3) =  = 3.0769 = 

3.0769x4 – 3.0769x3 = x3 – 1 → 3.0769x4 – 4.0769x3 + 1 = 0

C. defender 2 challenger 4 differences in transactions

-2000 850 850 850

IRR = 13.2% < MARR Therefore option 2 remains the defender (option 4 is rejected)

D. defender 2 challenger 5 differences in transactions

-3000 1400 1400 1400

IRR = 18.9% > MARR Therefore option 5 becomes the defender (option 2 is rejected)

E. defender 5 challenger 6 differences in transactions

-1000 265 265 265

IRR = -10.6% < MARR Therefore option 5 remains the defender (option 6 is rejected)

F. defender 5 challenger 7 differences in transactions

-1000 325 325 325

IRR = -1.3% < MARR Therefore option 5 remains the defender (option 7 is rejected)

Conclusion: the best choice is option 5, which means that we invest in Plans I and III.

**(Note** the IRR method only applies if the PW profile has a standard form**)**