Credit Risk: Individual Loan Risk

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1. Why is credit risk analysis an important component of bank risk management? What recent activities by FIs have made the task of credit risk assessment more difficult for both bank managers and regulators?

Credit risk management is important for bank managers because it determines several features of a loan: interest rate, maturity, collateral and other covenants. Riskier projects require more analysis before loans are approved. If credit risk analysis is inadequate, default rates could be higher and push a bank into insolvency, especially if the markets are competitive and the margins are low.

Credit risk management has become more complicated over time because of the increase in off-balance-sheet activities that create implicit contracts and obligations between prospective lenders and buyers. Credit risks of some off-balance-sheet products such as loan commitments, options, and interest rate swaps, are difficult to assess because the contingent payoffs are not deterministic, making the pricing of these products complicated.

1. Differentiate between a secured and an unsecured loan. Who bears most of the risk in a fixed-rate loan? Why would bankers prefer to charge floating rates, especially for longer-maturity loans?

A secured loan is backed by some of the collateral that is pledged to the lender in the event of default. A lender has rights to the collateral, which can be liquidated to pay all or part of the loan. In a fixed-rate loan, the lender of the loan bears the risk of interest rate changes; if interest rates rise, the opportunity cost of lending is higher. If interest rates fall, then the lender benefits. Since it is harder to predict longer-term rates, FIs prefer to charge floating rates for longer-term bonds and pass the risks on to the borrower.

3. How does a spot loan differ from a loan commitment? What are the advantages and disadvantages of borrowing through a loan commitment?

A spot loan involves the immediate takedown of the loan amount by the borrower, while a loan commitment allows a borrower the option to take down the loan any time during a fixed period at a predetermined rate. This can be advantageous during periods of rising rates in that the borrower can borrow as needed at a predetermined rate. If the rates decline, the borrower can borrow from other sources. The disadvantage is the cost: an up-front fee is required in addition to a back-end fee for the unused portion of the commitment.

1. Why is commercial lending declining in importance in the U.S.? What effect does the decline have on overall commercial lending activities?

Commercial bank lending has been declining in importance because of disintermediation, a process in which customers are able to access financial markets directly such as in issuing commercial paper. The total amount of commercial paper outstanding in the U.S. has grown dramatically over the last decade. Historically, only the most creditworthy borrowers had access the commercial paper market, but more middle-market firms and financial institutions now have access to this market. As a consequence of this growth, the pool of borrowers available to bankers has become smaller and riskier. This makes the credit assessment and monitoring of loans more difficult.

5. What are the primary characteristics of residential mortgage loans? Why does the ratio of adjustable rate mortgages to fixed-rate mortgages in the economy vary over the interest rate cycle? When would the ratio be highest?

Residential mortgages contracts differ in size, the ratio of the loan amount to the value of the property, the maturity of the loan, the rate of interest of the loan, and whether the interest rate is fixed or adjustable. In addition, mortgage agreements differ in the amount of fees, commissions, discounts, and points that are paid by the borrower.

The ratio of adjustable rate mortgages to fixed-rate mortgages is lowest when interest rates are low because borrowers prefer to lock in the low market rates for long periods of time. When rates are high, the adjustable rate mortgages allow borrowers the potential to realize relief from high interest rates in the future when rates decline.

6. What are the three major classes of consumer loans at U.S. banks? How do revolving loans differ from automobile and other consumer installment loans?

Consumer loans can be classified as revolving loans, automobile loans, and other consumer loans that typically include fixed-term personal loans. Automobile loans and fixed-term personal loans usually have a maturity date at which time the loan is expected to have a zero balance. Revolving loans usually involve credit card debt, and as a result the balance will rise and fall as borrowers make payments and utilize the accounts. These accounts typically have maturities of 1 to 3 years, but the accounts normally are renewed if the payment history is satisfactory.

7. How does the credit card transaction process assist in the credit monitoring function of financial institutions? Which major parties receive a fee in the typical credit card transaction? Do the services provided warrant the payment of these associated fees?

Credit card transactions typically must be authorized by the cardholder’s bank. Thus verification of satisfactory credit quality occurs with each transaction. During the transaction process, fixed fees are charged to the merchant, the merchant’s bank, and the card issuer. The fees cover the data processing and technology services necessary to ensure that the revolving credit transaction process is accomplished.

8. What are compensating balances? What is the relationship between the amount of compensating balance requirement and the return on the loan to the FI?

A compensating balance is the portion of a loan that a borrower must keep on deposit with the credit-granting depository FI. Thus the funds are not available for use by the borrower. As the amount of compensating balance for a given loan size increases, the effective return on the loan increases for the lending institution.

9. County Bank offers one-year loans with a stated rate of 9 percent but requires a compensating balance of 10 percent. What is the true cost of this loan to the borrower? How does the cost change if the compensating balance is 15 percent? If the compensating balance is 20 percent?

The true cost is the loan rate ÷ (1 – compensating balance rate) = 9% ÷ (1.0 – 0.1) = 10 percent. For compensating balance rates of 15 percent and 20 percent, the true cost of the loan would be 10.59 percent and 11.25 percent respectively. Note that as the compensating balance requirement increases at a constant rate, the true cost of the loan increases at an increasing rate.

10. Metrobank offers one-year loans with a 9 percent stated or base rate, charges a 0.25 percent loan origination fee, imposes a 10 percent compensating balance requirement, and must pay a 6 percent reserve requirement to the Federal Reserve. The loans typically are repaid at maturity.

a. If the risk premium for a given customer is 2.5 percent, what is the simple promised interest return on the loan?

The simple promised interest return on the loan is L + m = 0.09 + 0.025 = 0.115 or 11.5 percent.

b. What is the contractually promised gross return on the loan per dollar lent?



c. Which of the fee items has the greatest impact on the gross return?

The compensating balance has the strongest effect on the gross return on the loan. Without the compensating balance, the gross return would equal 11.75 percent, a reduction of 1.22 percent. Without the origination fee, the gross return would be 12.69 percent, a reduction of only 0.28 percent. Eliminating the reserve requirement would cause the gross return to increase to 13.06 percent, an increase of 0.09 percent.

11. Why are most retail borrowers charged the same rate of interest, implying the same risk premium or class? What is credit rationing? How is it used to control credit risks with respect to retail and wholesale loans?

Most retail loans are small in size relative to the overall investment portfolio of an FI, and the cost of collecting information on household borrowers is high. As a result, most retail borrowers are charged the same rate of interest that implies the same level of risk.

Credit rationing involves restricting the amount of loans that are available to individual borrowers. On the retail side, the amount of loans provided to borrowers may be determined solely by the proportion of loans desired in this category rather than price or interest rate differences, thus the actual credit quality of the individual borrowers. On the wholesale side, the FI may use both credit quantity and interest rates to control credit risk. Typically more risky borrowers are charged a higher risk premium to control credit risk. However, the expected returns from increasingly higher interest rates that reflect higher credit risk at some point will be offset by higher default rates. Thus rationing credit through quantity limits will occur at some interest rate level even though positive loan demand exists at even higher risk premiums.

12. Why could a lender’s expected return be lower when the risk premium is increased on a loan? In addition to the risk premium, how can a lender increase the expected return on a wholesale loan? A retail loan?

An increase in risk premiums indicates a riskier pool of clients who are more likely to default by taking on riskier projects. This reduces the repayment probability and lowers the expected return to the lender. In both cases the lender often is able to charge fees that increase the return on the loan. However, in both cases also, the fees may become sufficiently high as to increase the risk of nonpayment of default on the loan.

13. What are covenants in a loan agreement? What are the objectives of covenants? How can these covenants be negative? Affirmative?

Covenants are restrictions that are written into loan or bond contracts that affect the actions of the borrower. Negative covenants in effect restrict actions, that is, they are “thou shall not...” conditions. Common examples include the nonincrease of dividend payments without permission of the borrower, or the maintenance of net working capital above some minimum level. Positive covenants encourage actions such as the submission of quarterly financial statements. In effect both types of covenants are designed and implemented to assist the lending firm in the monitoring and control of credit risk.

14. Identify and define the borrower-specific and market-specific factors which enter into the credit decision. What is the impact of each factor on the risk premium?

The borrower-specific factors are:

Reputation: Based on the lending history of the borrower; better reputation implies lower risk premium.

Leverage: A measure of the existing debt of the borrower; the larger the debt, the higher the risk premium.

Volatility of earnings: The more stable the earnings, the lower the risk premium.

Collateral: If collateral is offered, the risk premium is lower.

Market-specific factors include:

Business cycle: Lenders are less likely to lend if a recession is forecasted.

Level of interest rates: A higher level of interest rates may lead to higher default rates, so lenders are more reluctant to lend under such conditions.

a. Which of these factors is more likely to affect adversely small businesses rather than large businesses in the credit assessment process by lenders?

Because reputation involves a history of performance over an extended time period, small businesses that are fairly young in operating time may suffer.

b. How does the existence of a high debt ratio typically affect the risk of the borrower? Is it possible that high leverage may reduce the risk of bankruptcy (or the risk of financial distress)? Explain.

Increasing amounts of debt increase the interest charges that must be paid by the borrower, and thus decrease the amount of cash flows available to repay the debt principal. Cases have been made that high debt levels require the firm to be very efficient in its managerial decision making, thus reducing the probability of bankruptcy.

c. Why is the volatility of the earnings stream of a borrower important to a lender?

A highly volatile earnings stream increases the probability that the borrower cannot meet the fixed interest and principal payments for any given capital structure.

15. Why is the degree of collateral as specified in the loan agreement of importance to the lender? If the book value of the collateral is greater than or equal to the amount of the loan, is the credit risk of the lender fully covered? Why, or why not?

Collateral provides the lender with some assets that can be used against the amount of the loan in the case of default. However, collateral has value only to the extent of its market value, and thus a loan fully collateralized at book value may not be fully collateralized at market value. Further, errors in the recording of collateralized positions may limit or severely reduce the protected positions of a lender.

16. Why are FIs consistently interested in the expected level of economic activity in the markets in which they operate? Why is monetary policy of the Federal Reserve System important to FIs?

During recessions firms in certain industries are much more likely to suffer financial distress because of the slowdown in economic activity. Specifically, the consumer durables industries are particularly hard hit because of cutbacks in spending by consumers. Fed monetary actions that increase interest rates cause FIs to sustain a higher cost of funds and cause borrowers to increase the risk of investments. The higher cost of funds to the FI can be passed along to the

borrower, but the increased risk in the investment portfolio necessary to generate returns to cover the higher funding cost to the borrower may lead to increased default risk realization. Thus actions by the Fed often are signals of future economic activity.

17. What are the purposes of credit scoring models? How could these models possibly assist an FI manager to better administer credit?

Credit scoring models are used to calculate the probability of default or to sort borrowers into different default risk classes. The models use data on observed economic and financial borrower characteristics to assist an FI manager in (a) identifying factors of importance in explaining default risk, (b) evaluating the relative degree of importance of these factors, (c) improving the pricing of default risk, (d) screening bad loan applicants, and (e) more efficiently calculating the necessary reserves to protect against future loan losses.

18. Suppose the estimated linear probability model is Z = 1.1X1 + .6X2 + .5X3 + error, where X1 = 0.75 is the borrower's debt/equity ratio; X2 = 0.25 is the volatility of borrower earnings; and X3 = 0.15 is the borrower’s profit ratio.

a. What is the projected probability of repayment for the borrower?

Z = 1.1(.75) + .6(.25) + .5(.15) = 1.05%.

The expected probability of repayment is 1 ‑ 0.0105 = 98.95%.

b. What is the projected probability of repayment if the debt/equity ratio is 3.5?

Z = 1.1(3.5) + .6(.25) + .5(.15) = 4.075%.

The expected probability of repayment is 1 ‑ 0.04075 = 95.925%.

c. What is a major weakness of the linear probability model?

A major weakness of this model is that the estimated probabilities can be below 0 or above 1.0, an occurrence that does not make economic or statistical sense.

19. Describe how a linear discriminant analysis model works. Identify and discuss the criticisms that have been made regarding the use of this type of model to make credit risk evaluations.

Linear discriminant models divide borrowers into high or low default classes contingent on their observed characteristics. The overall measure of default risk classification (Z) depends on the values of various financial ratios and the weighted importance of these ratios based on the past or observed experience. These weights are derived from a discriminant analysis model.

Several criticisms have been levied against these types of models. First, the models identify only two extreme categories of risk, default or no default. The real world considers several categories of default severity. Second, The relative weights of the variables may change over time. Further, the actual variables to be included in the model may change over time. Third, hard to define, but potentially important, qualitative variables are omitted from the analysis. Fourth, the real-world database of defaulted loans is very incomplete. Finally, the model is very sensitive to changes in variables. A change in sales of 40 percent may cause the model to provide different accept/reject decisions, but a decrease in sales in the real world normally is not seen as hard evidence that credit should be denied or withdrawn from an otherwise successful company.

20. MNO, Inc., a publicly traded manufacturing firm in the United States, has provided the following financial information in its application for a loan.

**Assets Liabilities and Equity**

Cash $ 20 Accounts Payable $ 30

Accounts Receivables $ 90 Notes Payable $ 90

Inventory $ 90 Accruals $ 30

Long Term Debt $150

Plant and equipment $500 Equity $400

Total Assets $700 Total Liabilities & Equity $700

Also assume sales = $500, cost of goods sold = $360, taxes = $56, interest payments = $40, net income = $44, the dividend payout ratio is 50 percent, and the market value of equity is equal to the book value.

a. What is the Altman discriminant function value for MNO, Inc.? Recall that:

Net working capital = current assets minus current liabilities.

Current assets = Cash + accounts receivable + inventories.

Current liabilities = Accounts payable + accruals + notes payable.

EBIT = Revenues ‑ Cost of goods sold ‑ depreciation.

Taxes = (EBIT ‑ Interest)(tax rate).

Net income = EBIT ‑ Interest ‑ Taxes.

Retained earnings = Net income (1 ‑ dividend payout ratio)

Altman’s discriminant function is given by: Z = *1.2X1* + *1.4X2* + *3.3X3* + *0.6X4* + *1.0X5*

Assume prior retained earnings are zero.

*X*1 = (200 ‑30 ‑30 -90)/ 700 = .0714 *X*1 = Working capital/total assets (TA)

*X*2 = 22 / 700 = .0314 *X*2 = Retained earnings/TA

*X*3 = 140 / 700 = .20 *X*3 = EBIT/TA

*X*4 = 400 / 150 = 2.67 *X*4 = Market value of equity/long term debt

*X5* = 500 / 700 = .7143 *X*5 = Sales/TA

Z = 1.2(0.07) + 1.4(0.03) + 3.3(0.20) + 0.6(2.67) + 1.0(0.71) = 3.104

= .0857 + .044 + .66 + 1.6 + .7143 = 3.104

b. Should you approve MNO, Inc.'s application to your bank for a $500,000 capital expansion loan?

Since the Z-score of 3.104 is greater than 1.81, ABC Inc.’s application for a capital expansion loan should be approved.

c. If sales for MNO were $300,000, the market value of equity was only half of book value, and the cost of goods sold and interest were unchanged, what would be the net income for MNO? Assume the tax credit can be used to offset other tax liabilities incurred by other divisions of the firm. Would your credit decision change?

ABC’s net income would be -$100 without taking into account text credits. Note, that ABC's tax liability is ‑$56,000. If we assume that ABC uses this tax credit against other tax liabilities, then:

*X*1 = (200 ‑ 30 ‑ 30 ‑ 90) / 700 = .0714

*X*2 = ‑44 / 700 = ‑0.0629

*X*3 = ‑60 / 700 = ‑0.0857

*X*4 = 200 / 150 = 1.3333

*X*5 = 300 / 700 = 0.4286

Since ABC's Z‑score falls to $.9434 < 1.81, credit should be denied.

d. Would the discriminant function change for firms in different industries? Would the function be different for retail lending in different geographic sections of the country? What are the implications for the use of these types of models by FIs?

The discriminant function models are very sensitive to the weights for the different variables. Since different industries have different operating characteristics, a reasonable answer would be affirmative with the condition that there is no reason that the functions could not be similar for different industries. In the retail market, the demographics of the market play a big role in the value of the weights. For example, credit card companies often evaluate different models for different areas of the country. Because of the sensitivity of the models, extreme care should be taken in the process of selecting the correct sample to validate the model for use.

21. Consider the coefficients of Altman’s Z-score. Can you tell by the size of the coefficients which ratio appears most important in assessing the creditworthiness of a loan applicant? Explain.

Although X3, or EBIT/total assets has the highest coefficient (3.3), it is not necessarily the most important variable. Since the value of X3 is likely to be small, the product of 3.3 and X3 may be quite small. For some firms, particularly those in the retail business, the asset turnover ratio, X5 may be quite large and the product of the X5 coefficient (1.0) and X5 may be substantially larger than the corresponding number for X3. Generally, the factor that adds most to the Z score varies from firm to firm and industry to industry.

22. If the rate of one-year T-Bills currently is 6 percent, what is the repayment probability for each of the following two securities? Assume that if the loan is defaulted, no payments are expected. What is the market-determined risk premium for the corresponding probability of default for each security?

a. 1-year AA rated bond yielding 9.5 percent?

Probability of repayment = *p* = (1 + I)/(1 + k)

For an AA-rated bond = (1 + .06)/ (1 + .095) = 0.968, or 96.8 percent

The market determined risk premium is 0.095 – 0.060 = 0.035 or 3.5 percent

b. 1-year BB rated bond yielding 13.5 percent?

Probability of repayment = *p* = (1 + I)/(1 + k)

For BB-rated bond = (1 + .06)/(1 + .135) = 93.39 percent

The market determined risk premium is 0.135 – 0.060 = 0.075 or 7.5 percent

23. A bank has made a loan charging a base lending rate of 10 percent. It expects a probability of default of 5 percent. If the loan is defaulted, it expects to recover 50 percent of its money through sale of its collateral. What is the expected return on this loan?

E(*r*) = p(1 + k) + (1 - p)(1 + k)(α ) where α is the percentage generated when the loan is defaulted. E(r) = .95(1 + .10) + .05(1 + .10)(.50) = 1.0450 + .0275 = 1.0725 - 1.0 = 7.25%

24. Assume a one-year T-Bill is currently yielding 5.5 percent, and a AAA-rated discount bond with similar maturity is yielding 8.5 percent.

a. If the expected recovery from collateral in the event of default is 50 percent of principal and interest, what is the probability of repayment of the AAA-rated bond? What is the probability of default?

p(1 + k) + γ (1 - p)(1 + k) = 1+I. Solve for the probability of repayment (p)*:*



Therefore the probability of default is 1.0 - .9447 = 0.0553 or 5.53 percent.

b. What is the probability of repayment of the AAA-rated bond if the expected recovery from collateral in the case of default is 94.47 percent of principal and interest? What is the probability of default?



Therefore the probability of default is 1.0 – 0.5000 = 0.5000 or 50.00 percent.

c. What is the relationship between the probability of default and the proportion of principal and interest that may be recovered in the case of default on the loan?

The proportion of the loan’s principal and interest that is collectible on default is a perfect substitute for the probability of repayment should such defaults occur.

25. What is meant by the phrase *marginal default probability*? How does this term differ from *cumulative default probability*? How are the two terms related?

Marginal default probability is the probability of default in the given time period, whereas cumulative default probability is the probability of default across several time periods. For example, the cumulative default probability across two time periods is given below, where (p) is the probability of nondefault in a given time period.

CP2 = 1 – (p1) (p2)

26. Calculate the term structure of default probabilities over three years using the following spot rates from the Treasury and corporate bond (pure discount) yield curves. Be sure to calculate both the annual marginal and the cumulative default probabilities.

Spot Spot Spot

1 year 2 year 3 year

Treasury Bonds 5.0% 6.1% 7.0%

BBB rated Bonds 7.0% 8.2% 9.3%

The notation used for implied forward rates is f12 = forward rate from period 1 to period 2.

Treasury Securities BBB Graded Debt

(1.061)2 = (1.05)(1 + f12 ) (1.082)2 = (1.07)(1 + f12 )

f12 = 7.21% f12 = 9.41%

(1.07)3 = (1.061)2(1 + f23 ) (1.093)3 = (1.082)2(1 + f23 )

f23 = 8.82% f23 = 11.53%

Using the implied forward rates, estimate the annual marginal probability of repayment:

p01(1.07) = 1.05 => p1 = 98.13 percent

p12(1.0941) = 1.0721 => p2 = 97.99 percent

p23 (1.1153) = 1.0882 => p3 = 97.57 percent

Using marginal probabilities, estimate the cumulative probability of default:

cp02 = 1 - (p1 )(p2 )

= 1 - (.9813)(.9799) = 3.84 percent

cp03  = 1 - (p1 )(p2 )(p3 )

= 1 - (.9813)(.9799)(.9757) = 6.18 percent

27. The bond equivalent yields for U.S. Treasury and A-rated corporate bonds with maturities of 93 and 175 days are given below:

Bond Maturities 93 days 175 days

U.S. Treasury 8.07% 8.11%

A-rated corporate 8.42% 8.66%

Spread 0.35% 0.55%

a. What are the implied forward rates for both an 82-day Treasury and an 82-day A-rated bond beginning in 93 days? Use daily compounding on a 365-day year basis.

. The forward rate, f, for the period 93 days to 175 days, or 82 days, for the Treasury is:

(1 + 0.0811)175/365 = (1 + 0.0807)93/365 (1 + *f* )82/365 ⇒ *f* = 8.16 percent

The forward rate, f, for the corporate bond for the 82-day period is:

(1 + 0.0866)175/365 = (1 + 0.0842)93/365 (1 + *f* )82/365 ⇒ *f* = 8.933%

b. What is the implied probability of default on A-rated bonds over the next 93 days? Over 175 days?

The probability of repayment of the 93-day A-rated bond is:

*p*(1 + 0.0842)93/365 = (1 + 0.0807)93/365 ⇒ *p* = 99.92 percent

Therefore, the probability of default is (1 - *p*) = (1 - .9992) = 0.0008 or 0.08 percent.

The probability of repayment of the 175-day A-rated bond is:

*p*(1 + 0.0866)175/365 = (1 +0.0811)175/365 ⇒ *p* = 99.76 percent

Therefore, the probability of default is (1 - *p*) = (1 - .9976) = 0.0024 or 0.24 percent.

c. What is the implied default probability on an 82-day A-rated bond to be issued in 93 days?

The probability of repayment of the A-rated bond for the period 93 days to 175 days, *p*, is:

*p* (1.08933)82/365 = (1 + 0.0816)82/365 ⇒ *p* = .9984, or 99.84 percent

Therefore, the probability of default is (1 - *p*) or 0.0016 or 0.16 percent.

28. What is the mortality rate of a bond or loan? What are some of the problems with using a mortality rate approach to determine the probability of default of a given bond issue?

Mortality rates reflect the historic default risk experience of a bond or a loan. One major problem is that the approach looks backward rather than forward in determining probabilities of default. Further, the estimates are sensitive to the time period of the analysis, the number of bond issues, and the sizes of the issues.

29. The following is a schedule of historical defaults (yearly and cumulative) experienced by an FI manager on a portfolio of commercial and mortgage loans.

Years after Issuance

Loan Type 1 Year 2 Years 3 Years 4 Years 5 Years

Commercial:

Annual default 0.00% \_\_\_\_\_\_ 0.50% \_\_\_\_\_\_ 0.30%

Cumulative default \_\_\_\_\_\_ 0.10% \_\_\_\_\_\_ 0.80% \_\_\_\_\_\_

Mortgage:

Annual default 0.10% 0.25% 0.60% \_\_\_\_\_\_ 0.80%

Cumulative default ­\_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_ 1.64% \_\_\_\_\_\_

a. Complete the blank spaces in the table.

Commercial: Annual default 0.00%, 0.10%, 0.50%, 0.20%, and 0.30%

Cumulative default: 0.00%, 0.10%, 0.60%, 0.80%, and 1.10%

Mortgage: Yearly default 0.10%, 0.25%, 0.60%, 0.70%, and 0.80%

Cumulative default 0.10%, 0.35%, 0.95%, 1.64%, and 2.43%

Note: The annual survival rate is pt = 1 – annual default rate, and the cumulative default rate for n = 4 of mortgages is 1 – (p1\* p2\* p3\* p4) = 1 – (0.999\*0.9975\*0.9940\*0.9930).

b. What are the probabilities that each type of loan will not be in default after 5 years?

The cumulative survival rate is = (1-mmr1)\*(1-mmr2)\*(1-mmr3)\*(1-mmr4)\*(1-mmr5) where mmr = marginal mortality rate

Commercial loan = (1-0.)\*(1-0.001)\*(1-0.005)\*(1-0.002)\*(1-0.003) = 0.989 or 98.9%.

Mortgage loan = (1-0.001)\*(1-0.0025)\*(1-0.006)\*(1-0.007)\*(1-0.008) = 0.9757 or 97.57%.

c. What is the measured difference between the cumulative default (mortality) rates for commercial and mortgage loans after four years?

Looking at the table, the cumulative rates of default in year 4 are 0.80% and 1.64%, respectively, for the commercial and mortgage loans. Another way of estimation is:

Cumulative mortality rate (CMR) = 1- (1 - mmr1)(1 - mmr2)(1 - mmr3)(1 - mmr4)

For commercial loan = 1- (1 - 0.0010)(1 - 0.0010)(1 - 0.0020)(1 - 0.0050)

= 1- .9920 = 0.0080 or 0.80 percent.

For mortgage loan = 1- (1 - 0.0010)(1 - 0.0025)(1 - 0.0060)(1 - 0.0070)

= 1- .98359 = 0.01641 or 1.641 percent.

The difference in cumulative default rates is 1.641 - .80 = .8410 percent.

30. The Table below shows the dollar amounts of outstanding bonds and corresponding default amounts for every year over the past five years. Note that the default figures are in millions while those outstanding are in billions. The outstanding figures reflect default amounts and bond redemptions.

Years after Issuance

Loan Type 1 Year 2 Years 3 Years 4 Years 5 Years

A-rated: Annual default (millions) 0 0 0 $ 1 $ 2

Outstanding (billions) $100 $95 $93 $91 $88

B-rated: Annual default (millions) 0 $ 1 $ 2 $ 3 $ 4

Outstanding (billions) $100 $94 $92 $89 $85

C-rated: Annual default (millions) $ 1 $ 3 $ 5 $ 5 $ 6

Outstanding (billions) $100 $97 $90 $85 $79

1. What are the annual and cumulative default rates of the above bonds?

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A-rated Bonds | |  |  |  |  |  |  |
|  | Millions | Millions | Annual | Survival = | Cumulative | % Cumulative |  |
| Year | Default | Balance | Default | 1 - An. Def. | Default Rate | Default Rate |  |
| 1 | 0 | 100,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |  |
| 2 | 0 | 95,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |  |
| 3 | 0 | 93,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |  |
| 4 | 1 | 91,000 | 0.000011 | 0.999989 | 0.000011 | 0.0011% |  |
| 5 | 2 | 88,000 | 0.000023 | 0.999977 | 0.000034 | 0.0034% |  |
|  | Where cumulative default for nth year = 1 - product of survival rates to that year. | | | | | | |
|  |  |  |  |  |  |  |  |
| B-rated Bonds | |  |  |  |  |  |  |
|  | Millions | Millions | Annual | Survival = | Cumulative | % Cumulative |  |
| Year | Default | Balance | Default | 1 - An. Def. | Default Rate | Default Rate |  |
| 1 | 0 | 100,000 | 0.000000 | 1.000000 | 0.000000 | 0.0000% |  |
| 2 | 1 | 94,000 | 0.000011 | 0.999989 | 0.000011 | 0.0011% |  |
| 3 | 2 | 92,000 | 0.000022 | 0.999978 | 0.000032 | 0.0032% |  |
| 4 | 3 | 89,000 | 0.000034 | 0.999966 | 0.000066 | 0.0066% |  |
| 5 | 4 | 85,000 | 0.000047 | 0.999953 | 0.000113 | 0.0113% |  |
|  |  |  |  |  |  |  |  |
| C-rated Bonds | |  |  |  |  |  |  |
|  | Millions | Millions | Annual | Survival = | Cumulative | % Cumulative |  |
| Year | Default | Balance | Default | 1 - An. Def. | Default Rate | Default Rate |  |
| 1 | 1 | 100,000 | 0.000010 | 0.999990 | 0.000010 | 0.0010% |  |
| 2 | 3 | 97,000 | 0.000031 | 0.999969 | 0.000041 | 0.0041% |  |
| 3 | 5 | 90,000 | 0.000056 | 0.999944 | 0.000096 | 0.0096% |  |
| 4 | 5 | 85,000 | 0.000059 | 0.999941 | 0.000155 | 0.0155% |  |
| 5 | 6 | 79,000 | 0.000076 | 0.999924 | 0.000231 | 0.0231% |  |

Years after Issuance

Bond Type 1 Year 2 Years 3 Years 4 Years 5 Years

A-rated: Yearly default 0% 0% 0% 0.0011% 0.0023%

Cumulative default 0% 0% 0% 0.0011% 0.0034%

B-rated: Yearly default 0% 0.0011% 0.0022% 0.0034% 0.0047%

Cumulative default 0% 0.0011% 0.0032% 0.0066% 0.0113%

C-rated: Yearly default 0.0010% 0.0031% 0.0056% 0.0059% 0.0076%

Cumulative default 0.0010% 0.0041% 0.0096% 0.0155% 0.0231%

Note: These percentage values seem very small. More reasonable values can be obtained by increasing the default dollar values by a factor of ten, or by decreasing the outstanding balance values by a factor of 0.10. Either case will give the same answers that are shown below. While the percentage numbers seem somewhat more reasonable, the true values of the problem are (a) that default rates are higher on lower rated assets, and (b) that the cumulative default rate involves more than the sum of the annual default rates.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| C-rated Bonds | |  | Test with 10x default. | |  |  |  |
|  | Millions | Millions | Annual | Survival = | Cumulative | % Cumulative |  |
| Year | Default | Balance | Default | 1 - An. Def. | Default Rate | Default Rate |  |
| 1 | 10 | 100,000 | 0.000100 | 0.999900 | 0.000100 | 0.0100% |  |
| 2 | 30 | 97,000 | 0.000309 | 0.999691 | 0.000409 | 0.0409% |  |
| 3 | 50 | 90,000 | 0.000556 | 0.999444 | 0.000965 | 0.0965% |  |
| 4 | 50 | 85,000 | 0.000588 | 0.999412 | 0.001552 | 0.1552% |  |
| 5 | 60 | 79,000 | 0.000759 | 0.999241 | 0.002311 | 0.2311% |  |
| More meaningful to use 0.10x balance, will get same result. | | | | | |  |  |

31. What is RAROC? How does this model use the concept of duration to measure the risk exposure of a loan? How is the expected change in the credit premium measured? What precisely is ΔL in the RAROC equation?

RAROC is a measure of expected loan income in the form of interest and fees relative to some measure of asset risk. The RAROC model uses the duration model formulation to measure the change in the value of the loan for given changes or shocks in credit quality. The change in credit quality (ΔR) is measured by finding the change in the spread in yields between Treasury bonds and bonds of the same risk class of the loan. The actual value chosen is the highest change in yield spread for the same maturity or duration value assets. In this case, ΔL represents the change in loan value or the change in capital for the largest reasonable adverse changes in yield spreads. The actual equation for ΔL looks very similar to the duration equation.



32. A bank is planning to make a loan of $5,000,000 to a firm in the steel industry. It expects to charge an up-front fee of 1.5 percent and a servicing fee of 50 basis points. The loan has a maturity of 8 years and a duration of 7.5 years. The cost of funds (the RAROC benchmark) for the bank is 10 percent. Assume the bank has estimated the maximum change in the risk premium on the steel manufacturing sector to be approximately 4.2 percent, based on two years of historical data. The current market interest rate for loans in this sector is 12 percent.

a. Using the RAROC model, estimate whether the bank should make the loan?

RAROC = Fees and interest earned on loan/ Loan or capital risk

We ignore up-front fees in the calculation of the loan’s income.

Loan risk, or Δ*L* = -*D*L\**L*\*(ΔR/(1 + R) = = -7.5 \* $5m \* (.042/1.12) = -$1,406,250

Expected interest = 0.12 x $5,000,000 = $600,000

Servicing fees = 0.0050 x $5,000,000 = $25,000

Less cost of funds = 0.10 x $5,000,000 = -$500,000

Net interest and fee income = $125,000

RAROC = $125,000/1,406,250 = 8.89 percent. Since RAROC is lower than the cost of funds to the bank, the bank should not make the loan.

b. What should be the duration in order for this loan to be approved?

For RAROC to be 10 percent, loan risk should be:

$125,000/Δ*L* = 0.10 ⇒ Δ*L* = 125,000 / 0.10 = $1,250,000

⇒ -*DL* \* *L* \* (ΔR/(1 + R)) = 1,250,000

*DL* = 1,250,000/(5,000,000 \* (0.042/1.12)) = 6.67 years.

Thus, this loan can be made if the duration is reduced to 6.67 years from 7.5 years. The duration can be reduced.

c. Assuming that duration cannot be changed, how much additional interest and fee income would be necessary to make the loan acceptable?

Necessary RAROC = Income/Risk ⇒ Income = RAROC \* Risk

= $1,406,250 \*0.10 = $140,625

Therefore, additional income = $140,625 - $125,000 = $15,625.

d. Given the proposed income stream and the negotiated duration, what adjustment in the risk premium would be necessary to make the loan acceptable?

$125,000/0.10 = $1,250,000 ⇒ -$1,250,000 = -7.5\*$5,000,000\*(ΔR/1.12)

Thus ΔR = 1.12(-$1,250,000)/(-7.5\*$5,000,000) = 0.0373

33. A firm is issuing a two-year debt in the amount of $200,000. The current market value of the assets is $300,000. The risk-free rate is 6 percent, and the standard deviation of the rate of change in the underlying assets of the borrower is 10 percent. Using an options framework, determine the following:

a. The current market value of the loan.

b. The risk premium to be charged on the loan.

The following need to be estimated first: *d*, *h*1 and *h*2 .

*d* = *Be*-rt /*A* = $200,000*e*-.06(2) /300,000 = .5913 or 59.13 percent.

*h1* = -[0.5\*(.10)2 \*2 - l*n*(.5913)]/(.10)21/2 = -3.7863

*h2* = -[0.5\*(.10)2 \*2 + l*n*(.5913)]/(.10)21/2 = 3.6449

Current market value of loan = *l*(*t*) = *Be*-rt [*N*(*h*1)1/*d* + *N*(*h*2)]

= $177,384.09[1.6912 \* *N*(-3.7863) + *N*(3.6449)]

= $177,384.09[1.6912 \* 0.0001 + 0.9999] = $177,396.35

The risk premium k – I = (-1/t) ln[N(h2) + (1/d)N(h1)]

= (-½)ln[0.9999 + 1.6912\*0.0001] = 0.00035

34. A firm has assets of $200,000 and total debts of $175,000. Using an option pricing model, the implied volatility of the firm’s assets is estimated at $10,730. Under the KMV method, what is the expected default frequency (assuming a normal distribution for assets)?

The firm will be in technical bankruptcy if the value of the assets fall’s below $175,000. If σ = $10,730, then it takes 25,000/10,730 = 2.33 standard deviations for the assets to fall below this value. Under the assumption that the market value of the assets are normally distributed, then 2.33 represents a 1 percent probability that the firm will become bankrupt.

35. Carman County Bank (CCB) has outstanding a $5,000,000 face value, adjustable rate loan to a company that has a leverage ratio of 80 percent. The current risk free rate is 6 percent, and the time to maturity on the loan is exactly ½ year. The asset risk of the borrower, as measured by the standard deviation of the rate of change in the value of the underlying assets, is 12 percent. The normal density function values are given below:

h N(h) h N(h)

-2.55 0.0054 2.50 0.9938

-2.60 0.0047 2.55 0.9946

-2.65 0.0040 2.60 0.9953

-2.70 0.0035 2.65 0.9960

-2.75 0.0030 2.70 0.9965

a. Use the Merton option valuation model to determine the market value of the loan.

The following need to be estimated first: *d*, *h*1 and *h*2 .

*h1* = -[0.5\*(0.12)2\*0.5 - l*n*(0.8)]/(0.12)√0.5 = -0.226744/0.084853 = -2.672198

*h2* = -[0.5\*(0.12)2\*0.5 + l*n*(0.8)]/(0.12)√0.5 = 0.219544/0.084853 = 2.587346

Current market value of loan = *l*(*t*) = *Be*-rt [*N*(*h*1)1/*d* + *N*(*h*2)]

= $4,852,227.67[1.25\**N*(-2.672198) + *N*(2.587346)]

= $4,852,227.67 [1.25\*0.003778 + 0.995123]

= $4,851,478.00

b. What should be the interest rate for the last six months of the loan?

The risk premium k – I = (-1/t) ln[N(h2) + (1/d)N(h1)]

= (-1/0.5)ln[0.995123 + 1.25\*0.003778] = 0.000308

The loan rate = risk-free rate plus risk premium = 0.06 + 0.000308 = 0.060308 or 6.0308%.

The questions and problems that follow refer to Appendixes 11B and 11C.

36. From Table 11B-1, what is the probability of a loan upgrade? A loan downgrade?

The probability of an upgrade is 5.95% + 0.33% + 0.02% = 6.30%. The probability of a downgrade is 5.30% + 1.17% + 0.12% = 5.59%.

a. What is the impact of a rating upgrade or downgrade?

The effect of a rating upgrade or downgrade will be reflected on the credit-risk spreads or premiums on loans, and thus on the implied market value of the loan. A downgrade should cause this credit spread premium to rise.

b. How is the discount rate determined after a credit event has occurred?

The discount rate for each year in the future in which cash flows are expected to be received includes the forward rates from the current Treasury yield curve plus the annual credit spreads for loans of a particular rating class for each year. These credit spreads are determined by observing the spreads of the corporate bond market over Treasury securities.

c. Why does the probability distribution of possible loan values have a negative skew?

The negative skew occurs because the probability distribution is non-normal. The potential downside change in a loan’s value is greater than the possible upside change in value.

d. How do the capital requirements of the CreditMetrics approach differ from those of the BIS and Federal Reserve System?

The Fed and the BIS require the capital reserve to be 8 percent of the book value of the loan. Under CreditMetrics each loan is likely to have a different VAR and thus a different implied capital requirement. Further, this required capital is likely to be greater than 8 percent of book value because of the non-normality of the probability distributions.

37. A five-year fixed-rate loan of $100 million carries a 7 percent annual interest rate. The borrower is rated BB. Based on hypothetical historical data, the probability distribution given below has been determined for various ratings upgrades, downgrades, status quo, and default possibilities over the next year. Information also is presented reflecting the forward rates of the current Treasury yield curve and the annual credit spreads of the various maturities of BBB bonds over Treasuries.

New Loan

Probability Value plus Forward Rate Spreads at time t

Rating Distribution Coupon $ t rt% st%

AAA 0.01% $114.82 1 3.00% 0.72%

AA 0.31% $114.60 2 3.40% 0.96%

A 1.45% $114.03 3 3.75% 1.16%

BBB 6.05% 4 4.00% 1.30%

BB 85.48% $108.55

B 5.60% $98.43

CCC 0.90% $86.82

Default 0.20% $54.12

a. What is the present value of the loan at the end of the one-year risk horizon for the case where the borrower has been upgraded from BB to BBB?



b. What is the mean (expected) value of the loan at the end of year one?

The solution table on the following page reveals a value of $108.06.

c. What is the volatility of the loan value at the end of the year?

The volatility or standard deviation of the loan value is $4.19.

d. Calculate the 5 percent and 1 percent VARs for this loan assuming a normal distribution of values.

The 5 percent VAR is 1.65 x $4.19 = $6.91.

The 1 percent VAR is 2.33 x $4.19 = $9.76.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  | Probability |
|  | Year-end |  |  | Probability |  | \* Deviation |
|  | Rating | Probability | Value | \* Value | Deviation | Squared |
|  | AAA | 0.0001 | $114.82 | $0.01 | 6.76 | 0.0046 |
|  | AA | 0.0031 | $114.60 | $0.36 | 6.54 | 0.1325 |
|  | A | 0.0145 | $114.03 | $1.65 | 5.97 | 0.5162 |
|  | BBB | 0.0605 | $113.27 | $6.85 | 5.21 | 1.6402 |
|  | BB | 0.8548 | $108.55 | $92.79 | 0.49 | 0.2025 |
|  | B | 0.056 | $98.43 | $5.51 | -9.63 | 5.1968 |
|  | CCC | 0.009 | $86.82 | $0.78 | -21.24 | 4.0615 |
|  | Default | 0.002 | $54.12 | $0.11 | -53.94 | 5.8197 |
|  |  | 1.000 | Mean = | $108.06 | Variance = | 17.5740 |
|  |  |  |  | Standard Deviation = | | $4.19 |

e. Estimate the “approximate” 5 percent and 1 percent VARs using the actual distribution of loan values and probabilities.

5% VAR = 95% of actual distribution = $108.06 - $102.02 = $6.04

1% VAR = 99% of actual distribution = $108.06 - $86.82 = $21.24

where: 5% VAR is approximated by 0.056 + 0.009 + 0.002 = 0.067 or 6.7 percent, and

1% VAR is approximated by 0.009 + 0.002 = 0.011 or 1.1 percent.

Using linear interpolation, the 5% VAR = $10.65 million and the 1% VAR = $19.31 million. For the 1% VAR, $19.31 = (1 – 0.1/1.1)\*$21.24.

f. How do the capital requirements of the 1 percent VARs calculated in parts (d) and (e) above compare with the capital requirements of the BIS and Federal Reserve System?

The Fed and BIS systems would require 8 percent of the loan value, or $8 million. The 1 percent VAR would require $19.31 million under the approximate method, and $9.76 million in capital under the normal distribution assumption. In each case, the amounts exceed the Fed/BIS amount.

38. How does the Credit Risk+ model of Credit Suisse Financial Products differ from the CreditMetrics model of J.P. Morgan?

Credit Risk attempts to estimate the expected loss of loans and the distribution of these losses with the focus on calculating the required capital reserves necessary to meet these losses. The method assumes that the probability of any individual loan defaulting is random, and that the correlation between the defaults on any pair of loan defaults is zero. CreditMetrics is focussed on estimating a complete VAR framework.

39. An FI has a loan portfolio of 10,000 loans of $10,000 each. The loans have an historical default rate of 4 percent, and the severity of loss is 40 cents per $1.

a. Over the next year, what are the probabilities of having default rates of 2, 3, 4, 5, and 8 percent?

 

n 2 3 4 5 8

Probability 0.1465 0.1954 0.1954 0.1563 0.0298

b. What would be the dollar loss on the portfolios with default rates of 4 and 8 percent?

Dollar loss of 4 loans defaulting = 4 x 0.40 x $10,000 = $16,000

Dollar loss of 8 loans defaulting = 8 x 0.40 x $10,000 = $32,000

c. How much capital would need to be reserved to meet the 1 percent worst-case loss scenario? What proportion of the portfolio’s value would this capital reserve be?

The probability of 8 defaults is ~3 percent. The probability of 10 defaults is 0.0106 or close to 1 percent. The dollar loss of 10 loans defaulting is $40,000. Thus a 1 percent chance of losing $40,000 exists.

A capital reserve should be held to meet the difference between the unexpected 1 percent loss rate and the expected loss rate of 4 defaults. This difference is $40,000 minus $16,000 or $24,000. This amount is 0.024 percent of the total portfolio.

1. Go to the Federal Reserve Board’s web site and update Table 11-1.

The answer will depend on the date of the assignment. The web site is <http://www.federalreserve.gov/>. Click on “Research and Data.” Click on “Statistics: Releases and Historical Data.” Click on “Assets and Liabilities of Commercial Banks of the United States, Releases.” Click on the most recent date. This will bring the file onto your computer that contains the relevant data.

1. Go to the Federal Housing Finance Board’s web site and find the most recent data on the percentage of conventional single-family mortgages with adjustable rates.

The answer will depend on the date of the assignment. The web site is <http://www.fhfb.gov/>. Click on “Monthly Interest Rate Summary.” Click on “Periodic Summary Tables.” Click on “Percentage of Conventional Single Family Mortgages Originated by Major Lenders with Adjustable Rates.” This will bring the file onto your computer that contains the relevant data.

1. Go to the Federal Reserve board’s web site and update Table 11-7.

The answer will depend on the date of the assignment. The web site is <http://www.federalreserve.gov/>. Click on “Research and Data.” Click on “Statistics: Releases and Historical Data.” Click on “Consumer Credit, Releases.” Click on the most recent date. This will bring the file onto your computer that contains the relevant data.