**KINEMATICS OF PARTICLES**

***11.28*** a = ksin 

(a) 

v = 

v = 

v = 



s = 

s = 

s = k

1. v = vmax  when 

i.e. when t = T

vmax = 2

1. S/t=2T = 

= 

= 

S = 



= 

S/t=2T = 

1. vave = 

Vave = 

A small object is released from rest in a tank of oil. The downward acceleration of the object

Is (g-kv) where g is the constant of the object is (g-kv) where g is the constant acceleration due to gravity, k is a constant which depends on the viscosity of the oil and the shape of the object and v is the downward velocity of the object. Derive expressions for the velocity, v, and the vertical drop, y, as function of the time, t, after release.

a =  (separation of variables)

Let (g-kv) = u ⇒ -kd0v=du

* 

ln 

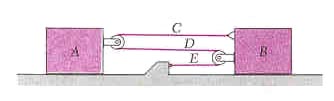
V = g 

v = 

y = 

Y=

***11.42***



xb

xo

e

xA

xE

xD

xP

1

T

Consider rope TCD: (assume rope inextensible)







XB = XP +

⇒

1. 
2. 
3. 
4. 

Determine the relationship which governs the velocities of the four cylinders. Express all velocities as positive down. How many degrees of freedom are there?

# A

# D

# B

# C

XC

XF

II

XE

I

XD

III

XA

Xt

E

F

XB

Assumption: Ropes I, II & III are inextensible

Rope I: XC + 2XE = Constant(i)

Rope II: (XC-/XE) + XC + 2/XF = Constant (ii)

Rope III: (XB-XF) +XB +XA = Constant (iii)

Substitute (i) in (ii) ⇒ 2XC + 2XF += Constant (iv)

Substitute (iv) in (iii) ⇒ 2XB + XA + XC + 

or 4XA +8XB +4XC +XD = Constant (v)

Differentiate (v) with respect to t

* 

OR

4VA + 8VB + 4VC +VD = 0

Only 3 of XA, XB, XC and XD in (V) can be chosen ⇒ system has 3 degrees of freedom

A ball is dropped vertically onto a 20° incline at A; the direction of rebound forms an angle of 40° with the vertical. Knowing that the ball next strikes the incline at B, determine

1. the velocity of rebound at A, (b) the time required for the ball to travel from A to B

Y

3m

40o

A

B

X

20o

* Motion in the x-direction



* Motion in the Y-direction



* Geometry:



1. then becomes: -1.092 = V0sin50°.

-1.092 = 3 tan 50° - 4.5

T = 0.975secs

V0 = 4.785 

1. (b)

Calculate the minimum possible magnitude of the muzzle velocity, u, which a projectile must have when fixed from point A to reach a target B on the same horizontal plane 12 km away.

12 km

θ

A

Y

B

X

* Motion in the X=direction



⇒12000 = ucosθ.t (i)

* Motion in the Y-direction



* At maximum height, VY = 0 ⇒ 0 usinθ-gt1⇒t1= (iii)
* Because of symmetry, time taken, t, to travel from A to is twice t1 (t = 2t1)

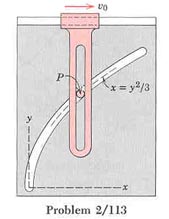
From (ii) ⇒ 12000 = ucos

u2 = 

⇒

(negative value for u not physically possible)

Umin = ± 



The vertical slot moves to the right with a constant speed *v*0 in meters per second for an interval of motion and causes the pin *P* to move along the parabolic slot *x* = *y*2/3 where *x* and *y* are in meters. Calculate the radius of curvature ρ to the path for the position where *y* = 2 m and calculate the tangential acceleration *at* of the pin when it passes this position. *Ans.* ρ = 6.94 m, *at =* -0.169*v*02 m/s2

For *y* = 2m, ⇒ *x* = 4/3 m (since *x* = *y*2/3)

t

X = 4/3m

Y

θ

V

y = 1m

x

y = 2m

x = v0

*n*

*an*

 ⇒





At Y = 2m, X = 



V2 = 

an = 

3

4

θ

5

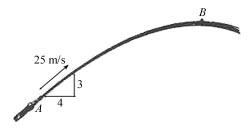






6.94 m/s2

m/s2



A nozzle discharges a stream of water in the direction shown with an initial velocity of 25 m/s. Determine the radius of curvature of the stream (*a*) as it leaves the nozzle, (*b*) at the maximum height of the stream.

At A



*g*

*tα*

*α*

*at*

*an*



At B

VB = (VA)X (Since ax = 0 and B is at the maximum)

# Height of the stream

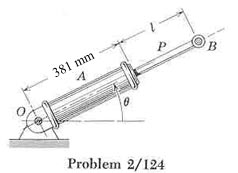
*an*

•

VB = VAcos = 25 (4/5) = 20 m/s





As the hydraulic cylinder rotates about 0 the length, l, of the piston rod P is controlled by the action of oil pressure in the cylinder. If the cylinder rotates at the constant rate = 60 deg/sec and is decreasing at the constant rate of 152.4 mm/sec, calculate the magnitudes of the velocity and acceleration of end B when l = 127 mag.

Solution

O

θ1

T

θ1

B

T1

r = 381 + 127 = 508 mm





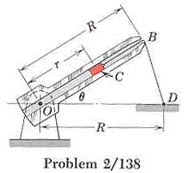
Vt = 



ar = 

a = 



****

The slotted arm is pivoted at *O* and carries the slider *C*. The position of *C* in the slot is governed by the cord which is fastened at *D* and remains taut. The arm turns counterclockwise with a constant angular rate θ = 4rad/sec during an interval of its motion. The length *DBC* of the cord equals *R*, which makes *r* = 0 when θ = 0. Determine the magnitude of the acceleration of the slider at the position for which θ = 30o. The distance *R* is 15 in. *Ans*. *A* = 489 in./sec2

Solution

Geometry: 

C

B

θ

D

O

R

r

θ1

t1

R

From triangle OBD, 

Kinematics: ac = t1 + 1

r = 2Rsin

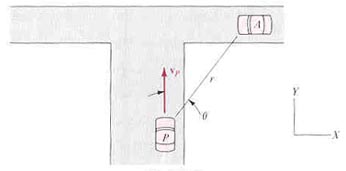
ac = 

= 30° ⇒ /2 = 15°, R = 15", 

r = 30 sin 15° = 7.76"





A police car *P* is traveling at velocity vP = 20j km/h when its radar gives the following data about car *A* : r = 100 m, = -60 km/h (r decreasing), ≈ 0, θ = 0.5 rad, = -10-3 rad/s (θ decreasing), and ≈ 0. Determine the velocity v*A*.

Solution:

* Relationship between the unit vectors *i*, *j* & *r*1, *θ*1

θ

VP

P

*r*

A

θ

*r*1

y, *j*

x, *i*

θ

θ1

θ

Note: θ1 is positive in the direction of increasing θ

*r*1 = *i* sinθ + j cosθ

θ1 = *I* cosθ - *j* sinθ

Velocity car *A* relative to the Policecar:



Absolute velocity of car *A*: 



### Geometry of a space curve

*S*

O

B

A

*T(s)*

Δ*I*

Δ*s*

*t*1

*n*1

*m*1

*T*(*s*+Δ*s*)

Consider the following space curve along which a particle moves from A to B

* Unit vector tangent to the curve at A is t1

t1 = lim



Δs→ 0

* Unit vector perpendicular to t1 and pointing toward centre of curvature of the curve is the principal normal n1
* Binotmal unit vector m1 = = t1 x n1
* -Plane of t1 & n1 : OSCULATING PLANE
* -Plane of m1 & n1: NORMAL PLANE
* -Plane of m1 & t1: RECTIFYING PLANE
* Consider segment AB of curve is more detail

Osculating plane

t1 (s+Δs)

T1 (s)

A

Δt1

B

t1 (s+Δs)

* The orientation of n1 (i.e. principal normal) is determined from the following:

1. n1 1 t1 (ii) n1 lies in the osculating plane (iii) 1n11 = 1

t1 .t1 = 1 ⇒ 

⇒ t1 1

The vector  Lim



Lies in the osculating plane since

Δs→ 0

Δt1 =t1 =t1 (S + S) -t1 (S) lies in the same plane. Therefore, the vector  satisfies conditions (i) and (iii) above; hence



Osculating Plane

X

Cx

δ

## A

X

P

R

Z

θ

φ

r

Y

C: Centre of curvature

: radius of curvature

V

P: Osculating plane

Consider the space curvilinear motion of a particle along the path shown. The motion at position A may be considered to be taking place in the plane P which contains the path at this position. This plane, often called the OSCULATING PLANE, may be defined by point A and two adjacent points, one on either side of A, on the path curve. As these points are brought closer to A, the plane containing the three points approaches the limiting plane P.

The osculating plane is, by definition, the plane containing the unit tangential (t1) and unit normal (n1) vectors. The path coordinates t and n, are used for solving most of the problems of planar motion. The osculating plane thus remains fixed in the plane of motion. Although it is possible to obtain the principal normal n and the direction of the osculating plane for spatial motion, it is generally easier to define the spatial motion of a particle using other coordinate systems, e.g. X,Y,Z or T,, Z.