**KINETICS OF PARTICLES**

Knowing that the coefficient of friction is 0.30 at all surfaces of contact, determine (a) the acceleration of plate A, (b) the tension in the cable. (Neglect bearing friction in the pulley)

B

A

5kg

10kg

125N

•

⚫ Kinematics

B

A

xB

yt

*l*

xA

xt

•

 - Assume cable is inextensible and, therefore of constant length

 → xA + xB = constant

 aA = -aB ……………….……. (i)

⚫ Motion of Plate B

* Free-body diagram

 ΣFy = 0: NB – 49.05 = 0 → NB = 49.05 N

T

WB = 5(9.81)N = 49.05N

T

NB

FB

 Friction: FB = 0.3NB = 0.3(49.05) = 14.72 N

 ΣFx = MB aB: FB-T = 5 aB

 → T = 14.72 – 5aB ………………(ii)

⚫ Motion of Plate A

* Free-body diagram

 ΣFy = 0 : NA – 49.05 – 98.0 = 0

125N

NB = 49.05N

FB = 14.72N

NA

FA

T

WA = 10(9.81)N = 98.1N

 → NA = 147.15 N

 Friction: FA = 0.3 NA = 0.3(147.15) N

 FA = 44.15 N

 Fx = mAaA: 125 – T – 14.72 – 44.15 = 10aA………….(iii)

 With (ii), (iii) becomes 125 – (14.72 – 5aB) – 14.72 – 44.15 = 10aA

 51.41 + 5aB = 10A

 aB = -aA from (i) → 

 From (ii) 

(a) Disk about to slide to the right

* Free-body diagram

Y

60o

X

a

mg

F

N

 ΣFx = max: 0.2 N = m a cos 60 ………………… (i)

 ΣFy = may: mg – N = m a sin 60 ………………..(ii)

 Solve (i) and (ii) simultaneously → 

(b) Disk about to slide to the left

* Free-body diagram

N

mg

F

Y

60o

X

a

 ΣFx = max: 0.2 N = m a cos 60 ………………… (iii)

 ΣFy = may: N - mg = m a sin 60 ………………..(iv)

 Solve (iii) and (iv) simultaneously → 



In a manufacturing process, disks are moved from one elevation to another by the lifting arm shown; the coefficient of friction between a disk and the arm is 0.20. Determine the magnitude of the acceleration for which the disks slide on the arm, assuming the acceleration is directed (a) downward as shown, (b) upward.

For the pulley arrangement shown below, with μs = 0.25 and μk = 0.20, calculate the acceleration of each block and the tension in the cable. Neglect the small mass and friction of the pulleys.

Kinematics

⚫ Assume cable is inextensible

 → xA – 2yB = constant → 

 → aA = 2aB …….. (i)

Free-body diagrams

20g

T

T

•

**B**

T

N

60g

F

30o

**A**

⚫ Check if block A slips

 For no slipping, ΣFx = 0 : 60 g sin 30 – T – F = 0

 → F = 30(9.81) – T = 294.3 – T

 Also ΣFy = 0 : 2T – 20 g = 0 → T = 10(9.81) = 98.1 N

 → F = 294.3 – 98.1 = 196.2 N

 Friction: Fmax = μsN = 0.25 (60 g cos 30) = 127.4 N

 F > Fmax → A slips down inclined plane.

⚫ Motion of A

 ΣFx = mAaA : 60 g sin 30 - T – F = 60 aA → 294.3 – T – F = 60 aA …………(ii)

ΣFy = 0 (A does not move in the y direction): N – 60 g cos 30 = 0 → N = 509.7 N…(iii)

 Friction: F = μkN = 0.2(509.7) = 101.9 N ………(iv)

 with (iv), (ii) becomes 294.3 – T – 101.9 = 60 aA → 60 aA + T = 192.4 ……..(v)

⚫ Motion of B

 ΣFy = mBaB: 2 T – 20 g = 20 aB → T – 10 aB = 98.1 ………..(vi)

 Rearrange (i), (v) and (vi) →

 aA – 2aB + (0)T = 0

 60 aA + (0)aB + T = 192.4

 (0)aA – 10 aB + T = 98.1

 - solve the above using Cramer’s rule

 











A 30-kg crate rests on a 20-kg cart. The coefficient of static friction between the crate and cart is 0.25. If the crate is not to slip with respect to the cart, determine (*a*) the maximum allowable magnitude of P, (*b*) the corresponding acceleration of the cart.

Crate: (Assume slipping impending)

30 kg

20kg

P – F = 0

P = F

P

x

y

N

W = 30-kg

F = 0.25N

 ∑Fy = 0 ⇒ N = 30g

30 *a*

 = F = 0.25(30g) = 7.5g

 ∑FA = max ⇒ 7.5g = 30 *a*

*a* = 2.45 m/s2

 *a*x = 0.25g ⇒

50g

P

50 *a*

 = Crate & Cart

∑Fx = *ma* ⇒ P = 50(0.25g) = 12.5g

P = 122.6 N

Check on assumption of slipping

Fmax = μSN = 0.25 x 30g = 7.5g

P – F = 0 ⇒ F = 12.5g ⇒ F > Fmax ⇒ crate slips

A 2 kg ball revolves in a horizontal circle as shown at a constant speed of 1.5 m/s. Knowing that L = 600 mm, determine (a) the angle θ that the cord forms with the vertical, (b) the tension in the cord.

⚫ Free-body diagram of ball

x

y

θ

T

2g

 - ball does not move in the y-direction

 → ΣFy = 0: Tcosθ = 2(98.1) = 19.62 ………… (i)

 - Motion of ball in the x-direction

 ΣFx = max: 

 or Tsin2θ = 7.5 ……(ii) → T = 7.5/sin2θ

 substitute for T in (i) → 

 sin2θ = 1 – cos2θ → 7.5 cosθ = 19.62 – 19.62cos2θ

 

 

 

 

  → 

 

Block A slides along a plane inclined at an angle α to the horizontal. The block is initially at a height, h, above a point B. Determine the velocity of the block at B if it starts from rest.

α

W

A

*h*

B

•

•

Solution (i) (Case of no friction)

 - Free-body diagram of block

Principle of Work and Energy

NP

F = μNP

α

Y

X

 Ui→B =TB - Ti Ui→B (initial position) Ti = 0 (Block starts from rest)

 Ui→B =  → 

Solution (ii) (Friction considered)

 Block A does not move in direction ⊥ inclined plane → ΣFy = 0: NP – W cos α = 0

 Ui→B = 

 → 

 For α = 90 → cot α = 0 → V = 

as in solution (i)

 For μ = 0 , V = 

 For real values of V, 2gh 2ghμ cot α → 



A 20 kg package is projected up a 20 incline with an initial velocity of 12 m/s. The coefficient of friction between the incline and the package is 0.15. Determine (a) the maximum distance the package will move up the inclined plane; (b) the velocity of the package when it returns to its original position.

(a) Free-body diagram of block

Principle of Work and Energy

NA

F = μNA

196.2N

20o

Y

X

 UA→B = TB - TA

 TA = 

 At maximum distance up the incline VB = 0 → TB = 0

 UA→B = -(196.2 sin 20 + 0.15 NA)d

 (Force and displacement in opposite directions)

 Package does not move ⊥ to inclined plane → ΣFy = 0

 NA – 196.2 cos 20 = 0 → NA = 196.2 cos 20

 → -(196.2) (sin 20 + 0.15 cos 20)d = 0 – 1440

 

(b) Free-body diagram

NB

F = 0.15NB

196.2N

20o

Principle of Work and Energy

 UB→A = TA - TB

 → TA = UB→A → 

  → 